



K26P 1307

Reg. No. :

Name :

Second Semester M.Sc. Degree (C.B.S.S. – Supplementary)
Examination, April 2026
(2022 Admission)
MATHEMATICS
MAT 2C 09 : Foundations of Complex Analysis

Time : 3 Hours

Max. Marks : 80

PART – A

Attempt **any four** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

1. Prove that $e^{z+a} = e^z e^a$.
2. State Cauchy's Integral formula (Second version).
3. Does the function $f(z) = e^{1/z}$ has a removable singularity at $z = 0$? Justify your answer.
4. Using Residue theorem, evaluate $\int_0^\infty \frac{1}{1+x^2} dx$.
5. Let $G = \text{ann}(0; 0, R)$ and let f be analytic on G with Laurent series development $f(z) = \sum_{n=-\infty}^\infty a_n z^n$. Put $f_n(z) = \sum_{k=-\infty}^n a_k z^k$ and show that $f_n \rightarrow f$ in $C(G, \mathbb{C})$.
6. Prove the following : If $\{f_n\} \subset H(G)$ converges to f in $H(G)$ and each f_n never vanishes on G then either $f \equiv 0$ or f never vanishes.

PART – B

Answer **any four** questions from this Part without omitting any Unit. **Each** question carries **16** marks. **(4×16=64)**

Unit – I

7. a) Prove the following : Suppose f is analytic in $B(a, R)$ and let $\alpha = f(a)$. If $f(z) - \alpha$ has a zero of order m at $z = a$ then there is an $\epsilon > 0$ and $\delta > 0$ such that for $|\zeta - \alpha| < \delta$, the equation $f(z) = \zeta$ has exactly m simple roots in $B(a, \epsilon)$.
- b) State and Prove The Open Mapping Theorem.
- c) Prove the following : Suppose $f : G \rightarrow \mathbb{C}$ is one-one and $f(G) = \Omega$. Then $f^{-1} : \Omega \rightarrow G$ is analytic and $(f^{-1})'(w) = [f'(z)]^{-1}$ where $w = f(z)$.

P.T.O.



8. State and Prove the Third version of Cauchy's Theorem.
9. Prove the following : let G be a connected open set and let $f : G \rightarrow \mathbb{C}$ be an analytic function. Then the following conditions are equivalent.
- $f \equiv 0$;
 - there is a point a in G such that $f^n(a) = 0$ for each $n \geq 0$;
 - $\{z \in G : f(z) = 0\}$ has a limit point in G .

Unit – II

10. a) Show that $\int_0^\infty \frac{\log x}{1+x^2} dx = 0$.
- b) Suppose that f has a simple pole at $z = a$ and let g be analytic in an open set containing a . Show that $\text{Res}(fg, a) = g(a) \text{Res}(f, a)$.
11. State and prove The Laurent Series Development.
12. a) State and prove Maximum Modules Theorem (Third version).
- b) State and prove Schwarz's Lemma.

Unit – III

13. a) Prove the following : If G is open in $\mathbb{C}$ then there is a sequence $\{K_n\}$ of compact subsets of G such that $G = \bigcup_{n=1}^\infty K_n$. Moreover the sets K_n can be chosen to satisfy the following conditions :
- $K_n \subset \text{int} K_{n+1}$.
 - $K \subset G$ and K is compact implies $K \subset K_n$ for some n .
 - Every component of $G_\infty - K_n$ contains a component of $G_\infty - G$.
- b) With the usual notations, show that $(C(G, \Omega), \rho)$ is a metric space.



14. a) State and prove Montel's Theorem.

b) Discuss the convergence of the infinite product $\prod_{n=2}^{\infty} \left(1 - \frac{1}{n^2}\right)$.

15. a) Prove the following : Let G be a region which is not the whole plane and such that every non-vanishing analytic function on G has an analytic square root. If $a \in G$ then there is an analytic function f on G such that :

i) $f(a) = 0$ and $f'(a) > 0$.

ii) f is one-one.

iii) $f(G) = D = \{z : |z| < 1\}$.

b) Prove that $\lim_{z \rightarrow 0} \frac{\log(1+z)}{z} = 1$.

