



K26P 1214

Reg. No. :

Name :

**Second Semester M.Sc. Degree (C.B.C.S.S. – O.B.E. – Reg./Supple./Imp.)
Examination, April 2026
(2023 Admission Onwards)**

**MATHEMATICS/MATHEMATICS (MULTIVARIATE CALCULUS AND
MATHEMATICAL ANALYSIS, MODELLING AND SIMULATION, FINANCIAL
RISK MANAGEMENT)**

MSMAT02C06/MSMAF02C06 : Advanced Abstract Algebra

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **any 5** questions from this Part. **Each** question carries **4** marks.

1. Let D be a PID. Prove that every element that is neither 0 nor a unit in D is a product of irreducibles.
2. Use Euclidean algorithm to find a gcd of $8 + 6i$ and $5 - 15i$ in $\mathbb{Z}[i]$.
3. Let α be a root of $p(x) = x^2 + x + 1$ in an extension field of $\mathbb{Z}_2$. Show that $\{0, 1, \alpha, 1 + \alpha\}$ forms a field.
4. If $\alpha > 0$ is a constructible real number, then show that $\sqrt{\alpha}$ is constructible.
5. Show that if $[E : F] = 2$, then E is a splitting field over F .
6. Show that $\mathbb{Q}[\sqrt{2}, \sqrt{3}]$ is a separable extension of $\mathbb{Q}$. **(5×4=20)**

PART – B

Answer **any 3** questions from this Part. **Each** question carries **7** marks.

7. Let D be an integral domain with a multiplicative norm N .
 - a) Prove that $N(1) = 1$ and that $N(u) = 1$ for every unit u in D .
 - b) Suppose that every element $a \in D$ with $N(a) = 1$ is a unit. Show that if $n \in D$ satisfies $N(n) = p$, where p is a prime in $\mathbb{Z}$, then n is irreducible in D .
8. a) If F is a field, then prove that the polynomial ring $F[x]$ is a Euclidean domain.
 b) Show that $\sqrt{1 + \sqrt[3]{2}}$ is an algebraic number.

P.T.O.



9. Prove that every finite extension is an algebraic extension. Is the converse true ? Justify.
10. Prove that the splitting field E of $x^3 - 2$ over $\mathbb{Q}$ has degree 6 over $\mathbb{Q}$. Find a basis for the splitting field E over $\mathbb{Q}$.
11. State and prove "Primitive Element Theorem". (3×7=21)

PART – C

Answer **any 3** questions from this Part. **Each** question carries **13** marks.

12. Prove or disprove :
- a) In an integral domain a prime element is irreducible. 5
 - b) In an integral domain an irreducible element is prime. 8
13. a) Prove that product of any two primitive polynomials in $\mathbb{Z}[x]$ is a primitive polynomial in $\mathbb{Z}[x]$. 5
- b) Prove that the polynomial ring $\mathbb{Z}[x]$ is a UFD. 8
14. a) Describe all extensions of the identity map of $\mathbb{Q}$ to an isomorphism mapping $\mathbb{Q}(\sqrt{-5})$ onto a subfield of $\mathbb{C}$. 5
- b) Prove that for every prime power p^n , there exists a finite field $\text{GF}(p^n)$ with exactly p^n elements. 8
15. Let F be a field, and let α and β be algebraic over F with $\deg(\alpha, F) = n$. Define a map $\varphi_{\alpha, \beta} : F(\alpha) \rightarrow F(\beta)$ by $\varphi_{\alpha, \beta}(c_0 + c_1\alpha + \dots + c_{n-1}\alpha^{n-1}) = c_0 + c_1\beta + \dots + c_{n-1}\beta^{n-1}$, where $c_i \in F$. Prove that $\varphi_{\alpha, \beta}$ is an isomorphism of $F(\alpha)$ onto $F(\beta)$ if and only if α and β are conjugate over F .
16. a) Find the Galois group of $\mathbb{Q}[\sqrt{2}, \sqrt{5}]$ over $\mathbb{Q}$. 5
- b) If K is a finite extension of E and E is a finite extension of F , that is, $F \subseteq E \subseteq K$, then show that K is separable over F if and only if K is separable over E and E is separable over F . 8
- (3×13=39)**