K25P 3576

Reg. No. : .....

Name : .....

I Semester M.Sc. Degree (C.B.C.S.S. – OBE-Reg./Supple./Imp.) Examination, October 2025
(2023 Admission Onwards)
MATHEMATICS/MATHEMATICS (MULTIVARIATE CALCULUS AND MATHEMATICAL ANALYSIS, MODELLING AND SIMULATION, FINANCIAL RISK MANAGEMENT)
MSMAF01C01/MSMAT01C01 : Abstract Algebra

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks.

1. Find the order of (8, 4, 10) in the group $\mathbb{Z}_{12} \times \mathbb{Z}_{60} \times \mathbb{Z}_{24}$.
2. Let X be a G – set. Prove that $G_x = \{g \in G / gx = x\}$ is a subgroup of G for each $x \in G$.
3. Show that every group of order 45 has a normal subgroup of order 9.
4. Prove that the center of a finite nontrivial p-group G is normal.
5. State Zassenhaus lemma.
6. State Fermat's theorem. Find all zeros in $\mathbb{Z}_5$ of $2x^{219} + 3x^{74} + 2x^{57} + 3x^{44}$.

(5×4=20)

PART – B

Answer **any three** questions. **Each** question carries **7** marks.

7. Prove that the group $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic and isomorphic to $\mathbb{Z}_{mn}$ if and only if m and n are relatively prime.
8. Prove that no group of order 96 is simple.

P.T.O.

9. Prove that every abelian group of order 255 is abelian.

10. Let H be a subgroup of G and let N be a normal subgroup of G. Prove that $(HN)/N \simeq H/(H \cap N)$.

11. Let F be a field. Prove that every ideal in F[x] is a principal ideal. **(3×7=21)**

PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. a) State fundamental theorem of finitely generated abelian groups.

    b) Find all abelian groups of order 16 upto isomorphism.

    c) Find all subgroups of $\mathbb{Z}_2 \times \mathbb{Z}_4$ of order 4.

13. Let $G = D_4 = \{\rho_0, \rho_1, \rho_2, \rho_3, \mu_1, \mu_2, \delta_1, \delta_2\}$ be the group of symmetries of a square. Let $X = \{1, 2, 3, 4, s_1, s_2, s_3, s_4, m_1, m_2, d_1, d_2, C, P_1, P_2, P_3, P_4\}$, where 1, 2, 3, 4 are vertices of the square, s_1, s_2, s_3, s_4 are sides of the square, d_1, d_2 are diagonals of the square, m_1, m_2 are vertical and horizontal axes, C is the center point, P_i's are midpoints of the sides s_i. Give the table that describes completely the action of D_4 on X and find the orbits in X under D_4. Give the table that describes completely the action of D_4 on X and find $X_{\rho 0}$, $X_{\mu 1}$ and G_{d1}.

14. a) Let G be a finite group and p divides |G|. Prove that the number of sylow p subgroups is congruent to 1 modulo p and divides |G|.

    b) Find the conjugate classes and the class equation for S_4.

15. Let G be a nonzero free abelian group of finite rank n, and let K be a nonzero subgroup of G. Prove that K is free abelian of rank $s \leq n$ and furthermore, there exists a basis $\{x_1, x_2, ..., x_n\}$ for G and positive integers $d_1, d_2, ..., d_s$, where d_i divides d_{i+1} for $i = 1, 2, ..., s - 1$ such that $\{d_1x_1, d_2x_2, ..., d_sx_s\}$ is a basis for K.

16. a) Let R be a commutative ring with unity. Prove that M is a maximal ideal of R if and only if R/M is a field.

    b) Is $\mathbb{Q}[x]/\langle x^2 - 6x + 6 \rangle$ a field ? Why ? **(3×13=39)**

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