K25P 3577

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (C.B.C.S.S. – OBE-Reg./Supple./Imp.) Examination, October 2025
(2023 Admission Onwards)
MATHEMATICS/MATHEMATICS (MULTIVARIATE CALCULUS AND MATHEMATICAL ANALYSIS, MODELLING AND SIMULATION, FINANCIAL RISK MANAGEMENT)
MSMAF01C02/MSMAT01C02 : Linear Algebra

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks.

1. Find the null space and nullity of the identity transformation on R^2.
2. Let T be a linear operator on R^2 defined by $T(x_1, x_2) = (-x_2, x_1)$. Find the matrix of T with respect to the standard basis of R^2.
3. Define similar matrices. Prove that similar matrices have the same characteristic polynomial.
4. Find a 3×3 matrix for which the minimal polynomial is x^2.
5. Let T be a linear operator on R^3 with matrix representation $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. Prove that T has no cyclic vector.
6. Define orthogonal set. Let X be an inner product space. Prove that an orthonogonal subset of nonzero vectors of X is linearly independent. **(5×4=20)**

PART – B

Answer **any three** questions. **Each** question carries **7** marks.

7. Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. Suppose that V is finite dimensional. Prove that rank (T) + nullity (T) = dim V.
8. Prove that two finite dimensional vector spaces are isomorphic if and only if they have the same dimension.

P.T.O.

9. Let T be a linear operator on a finite dimensional space V and let c be a scalar. Prove that the following statements are equivalent :

   i) c is a characteristic value of T.

   ii) The operator (T – cI) is singular.

   iii) det (T – cI) = 0.

10. Find the minimal polynomial for $A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ -2 & -2 & 2 & 1 \\ 1 & 1 & -1 & 0 \end{bmatrix}$.

11. Define inner product. Verify that the standard inner product on F^n is an inner product. **(3×7=21)**

PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. Let V and W be finite dimensional vector spaces over F such that dim V = dim W. If T is a linear transformation from V into W, prove that the following conditions are equivalent.

   i) T is invertible.

   ii) T is nonsingular.

   iii) T is onto

   iv) If $\{\alpha_1, \alpha_2, ..., \alpha_n\}$ is a basis for V, then $\{T\alpha_1, T\alpha_2, ..., T\alpha_n\}$ is a basis for W.

   v) There is some basis $\{\alpha_1, \alpha_2, ..., \alpha_n\}$ for V, such that $\{T\alpha_1, T\alpha_2, ..., T\alpha_n\}$ is a basis for W.

13. Let W be the subspace of R^5 spanned by $\alpha_1 = (2, -2, 3, 4, -1)$, $\alpha_2 = (0, 0, -1, -2, 3)$, $\alpha_3 = (-1, 1, 2, 5, 2)$, $\alpha_4 = (1, -1, 2, 3, 0)$. Find a basis of the annihilator of W.

14. a) Let g, $f_1, ..., f_r$ be linear functionals on a vector space V with respective null spaces N, $N_1, ..., N_r$. Prove that g is a linear combination of $f_1, ..., f_r$ if and only if N contains the intersection $N_1 \cap ... \cap N_r$.

b) Let A be the real 3×3 matrix $\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$. Find characteristic values and corresponding characteristic vectors of A.

15. a) Let V be a finite dimensional vector space. Let $W_1, ..., W_k$ be subspace of V and let $W = W_1 + ... + W_k$. Prove that the following statements are equivalent :

i) $W_1, ..., W_k$ are independent.

ii) For each j, $2 \leq j \leq k$, we have $W_i \cap (W_1 + ... + W_{i-1}) = \{0\}$.

iii) If $\mathscr{B}_i$ is an ordered basis for W_i, $1 \leq i \leq k$, then the sequence $\mathscr{B} = (\mathscr{B}_i, ..., \mathscr{B}_k)$ is an ordered basis for W.

b) Let V be a finite dimensional vector space and let W_1 be any subspace of V. Prove that there is a subspace W_2 of V such that $V = W_1 \oplus W_2$.

16. State Gram Schmidt orthogonalization process and apply this process to the vectors $\beta_1 = (1, 0, 1)$, $\beta_2 = (1, 0, -1)$ and $\beta_3 = (0, 3, 4)$ to obtain an orthonormal basis for R^3 with the standard inner product. **(3×13=39)**