K25P 3580

Reg. No. : .....

Name : .....

I Semester M.Sc. Degree (C.B.C.S.S. – OBE-Reg./Supple./Imp.) Examination, October 2025
(2023 Admission Onwards)
MATHEMATICS/MATHEMATICS (MULTIVARIATE CALCULUS AND MATHEMATICAL/ANALYSIS, MODELLING AND SIMULATION, FINANCIAL RISK MANAGEMENT)
MSMAT01C04/MSMAF01C04 : Topology

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 5** questions from the following 6 questions. **Each** question carries **4** marks.

1. Let τ be the class consisting of $\mathbb{R}$, ϕ and all finite open intervals $A_q = (q, \infty)$ with $q \in \mathbb{Q}$, the rationals. Does τ is a topology on $\mathbb{R}$? Justify your answer.

2. Consider the following topology on X = {a, b, c, d, e} : τ = {X, ϕ , {a}, {a, b}, {a, c, d}, {a, b, c, d}, {a, b, e}}.

   i) List the closed subsets of X.

   ii) Find the closure of the sets {a}, {a, d}.

3. Prove that no countable subset of $\mathbb{R}$ is connected.

4. Show that the infinite open intervals (a, ∞) and $(-\infty, b)$, where $0 < a, b < 1$ form a subbase for the standard topology on $\mathbb{R}$.

5. Determine the smallest base $\mathcal{B}$ for the discrete topology on any non-empty set X.

6. Define a metrizable space. Give an example of a non metrizable space.

(5×4=20)

P.T.O.

PART – B

Answer **any 3** from the following 5 questions. **Each** question carries **7** marks.

7. Find the interior and closure of each of the following subsets of $\mathbb{R}^2$.
   a) $A = \{x \times y : y = 1\}$
   b) $B = \{x \times y : \quad x \text{ is rational}\}$
   c) $A \cup B$

8. Let $f, g : [0, 1] \to [0, 1]$ is continuous. Show that the set $\{x : f(x) \leq g(x)\}$ is closed in $[0, 1]$.

9. Which of the following are closed subspace of $\mathbb{R}^2$ with standard topology ? Justify your answer.
   a) $A = \{x \times y : xy = 1\}$
   b) $B = \{x \times y : |x| + |y| = 1\}$
   c) $C = \{x \times y : x^2 + y^2 < 1\}$

10. Which of the following are metrices on $\mathbb{N}$, the set of naturals ? Justify your answer.
   i) $d(x, y) = |x - y|$
   ii) $d(x, y) = \max\{x, y\}$
   iii) $d(x, y) = \dfrac{|x - y|}{2 + |x - y|}$

11. i) Show that $[0, 1]$ and $[0, 1/2) \cup (1/2, 1]$ are not homeomorphic to each other.
   ii) Show that $[-1, 1]$ and $[0, 1]$ are homeomorphic to each other. **(3×7=21)**

PART – C

Answer **any 3** from the following 5 questions. **Each** question carries **13** marks.

12. a) Prove the following :
   Let X be an ordered set in the order topology; let Y be a subset of X that is convex in X. Then the order topology on Y is the same as the topology which Y inherits as a subspace of X.
   b) Arbitrary intersection of closed sets are closed.
   c) Finite union of closed sets are closed.

13. Let X and Y are topological spaces; let $f : X \to Y$. Then prove that the following are equivalent.

    i) f is continuous.

    ii) For every subset A of X, $f(\bar{A}) \subset \overline{f(A)}$.

    iii) For every closed set B of X, the set $f^{-1}(B)$ is closed set in X.

    iv) For each $x \in X$ and each neighborhood V of f(x), there is a neighborhood U of x such that $f(U) \subset V$.

14. a) Show that $(X_1 \times X_2 \ldots \times X_{n-1}) \times X_n$ is homeomorphic with $(X_1 \times X_2 \ldots \times X_n)$.

    b) Prove the following : Let $f : A \to \Pi_{\alpha \in J} X_\alpha$ be given by the equation $f(a) = (f_\alpha(a))_{\alpha \in J}$, where $f_\alpha : A \to X_\alpha$ for each α. Let ΠX_α have the product topology. Then the function f is continuous if and only if each function f_α is continuous.

15. a) Show that the unit ball $B^n = \{x \in \mathbb{R}^n : ||x|| \leq 1\}$ is path connected.

    b) Give an example of a space which is connected but not path connected.

    c) Prove that every path connected space is connected.

16. a) Is a product of path connected spaces necessarily path connected ? Justify your answer.

    b) If $A \subset X$ and A is path connected, is $\bar{A}$ necessarily path connected ? Justify your answer.

    c) Let $X = \mathbb{R}^2 - \{x \times y : x, y \text{ are rationals}\}$. Is X is path connected ? Justify your answer. **(3×13=39)**

———————