K25P 3581

Reg. No. : .....

Name : .....

I Semester M.Sc. Degree (C.B.C.S.S. – OBE-Reg./Supple./Imp.) Examination, October 2025
(2023 Admission Onwards)
MATHEMATICS/MATHEMATICS (MULTIVARIATE CALCULUS AND MATHEMATICAL ANALYSIS, MODELLING AND SIMULATION, FINANCIAL RISK MANAGEMENT)
MSMAT01C05/MSMAF01C05 : Ordinary Differential Equations

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 5** from the following 6 questions. **Each** question carries **4** marks.

1. Using power series solution method find the binomial series expansion of the function $y = (1 + x)^p$ where p is an arbitrary constant.

2. Find the ordinary points and singular points of the differential equation $(1-x^2)y'' - xy' + \frac{5}{x}y = 0$.

3. Consider the generating function $\frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} P_n(x)t^n$ for the Legendre's polynomials. Using the generating function show that

   $(n + 1)P_{n+1}(x) = (2n + 1)\, xP_n(x) - nP_n(x)$.

4. Find the equivalent system of first order differential equations of the equation $y''' = y'' - x(y')^2$.

5. Show that $\lim_{p\to 0}\Gamma(p) = \pm\infty$.

6. Find the Wroskian of two solutions of the system of equations $\begin{cases} \frac{dx}{dt} = 2x \\ \frac{dy}{dt} = y \end{cases}$ **(5×4=20)**

P.T.O.

PART – B

Answer **any 3** from the following 5 questions. **Each** question carries **7** marks.

7. Consider the following differential equation $xy' = y$. Find a power series solution of the form $\sum a_n x^n$, solve the equation directly and explain any discrepancies that arise.

8. Find the general solution of $(1+x^2)y'' - 2xy' + 2y = 0$ in terms of power series in x. Can you express this solution by means of elementary functions ?

9. Solve the system of equations :

$$\begin{cases} \frac{dx}{dt} = 5x - 2y \\ \frac{dy}{dt} = 2x + y \end{cases}$$

10. Find the exact solution of the initial value problem $y' = 2x(1+y)$, $y(0) = 0$. Starting with $y_0(0) = 0$ calculate $y_1(x)$, $y_2(x)$ and $y_3(x)$ and compare these results with the exact solution.

11. Prove the following :

i) $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ ii) $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ **(3×7=21)**

PART – C

Answer **any 3** from the following 5 questions. **Each** question carries **13** marks.

12. Consider the equation $(1-x^2)y'' - xy' + p^2 y = 0$ where p is a constant.

    a) Find two linearly independent series solutions valid for $|x| < 1$.

    b) Show that if p = n where n is a non negative integer, then there is a polynomial solution of degree n.

13. a) With the usual notations, show that

$$P_n(x) = F\left(-n, n+1, 1, \frac{1}{2}(1-x)\right)$$

b) Prove the following :

i) $2J'_p(x) = J_{p-1}(x) - J_{p+1}(x)$

ii) $\frac{2p}{x} J_p(x) = J_{p-1}(x) + J_{p+1}(x)$

14. Prove the following :

a) If $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of $y'' + P(x)y + Q(x)y = 0$, then the zeros of these functions are distinct and occur alternately in the sense that $y_1(x)$ vanishes exactly once between any two successive zeros of $y_2(x)$ and conversely.

b) If $q(x) < 0$, and if $u(x)$ is a nontrivial solution of $u'' + q(x)u = 0$, then $u(x)$ has at most one zero.

15. a) Solve the following initial value problem by Picard's method, and compare the result with the exact solution :

$$\begin{cases} \frac{dy}{dx} = z, y(0) = 1, \\ \frac{dz}{dx} = -y, z(0) = 0 \end{cases}$$

b) Let P(x), Q(x) and R(x) be continuous functions on an interval $a \le x \le b$. If x_0 is any point in this interval and y_0 and y'_0 are any numbers whatever, then prove that the initial value problem $y'' + P(x)y' + Q(x) = R(x)$, $y(x_0) = y_0$, $y'(x_0) = y'_0$ has one and only one solution $y = y(x)$ on the interval $a \le x \le b$.

16. State and prove the orthogonality property of the Bessel functions. **(3×13=39)**

———————