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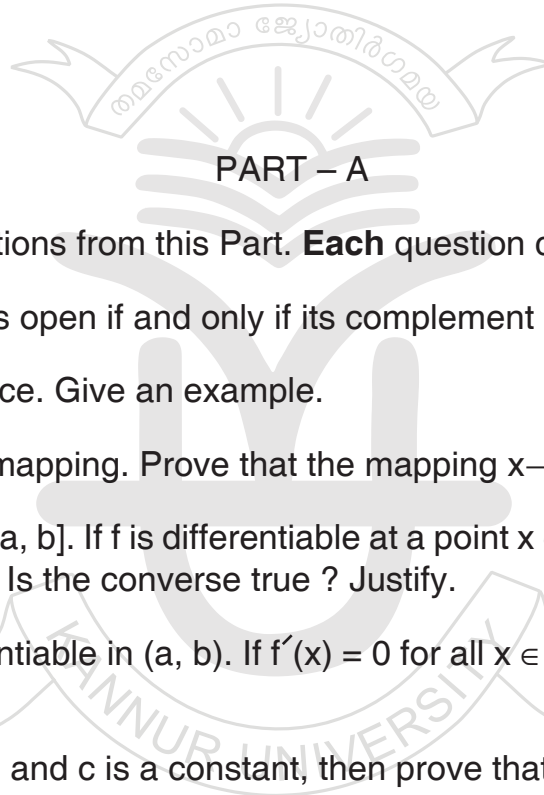
Reg. No. :

Name :

**I Semester M.Sc. Degree (CBCSS – OBE – Reg./Supple./Imp.)
Examination, October 2025
(2023 Admission Onwards)
MATHEMATICS
MSMAT01C03 : Real Analysis**

Time : 3 Hours

Max. Marks : 80



PART – A

Answer **any five** questions from this Part. **Each** question carries **4** marks. **(5×4=20)**

1. Prove that a set E is open if and only if its complement is closed.
2. Define a metric space. Give an example.
3. Define continuous mapping. Prove that the mapping $x \rightarrow |x|$ is continuous on \mathbb{R}^k .
4. Let f be defined on $[a, b]$. If f is differentiable at a point $x \in [a, b]$, then show that f is continuous at x . Is the converse true ? Justify.
5. Suppose f is differentiable in (a, b) . If $f'(x) = 0$ for all $x \in (a, b)$, then prove that f is a constant.
6. If $f_1 \in R(\alpha)$ on $[a, b]$, and c is a constant, then prove that $cf_1 \in R(\alpha)$ and
$$\int_a^b cf_1 d\alpha = c \int_a^b f_1 d\alpha .$$

PART – B

Answer **any three** questions from this Part. **Each** question carries **7** marks. **(3×7=21)**

7. Prove that the set of all rational numbers is countable.
8. Let f and g be complex continuous functions on a metric space X . Then prove that $f + g$, fg and $\frac{f}{g}$ are continuous on X .

P.T.O.



9. Give an example of a monotonic function f on (a, b) which is discontinuous at every point of a countable subset E of (a, b) and at no other point of (a, b) . Justify.
10. By giving suitable example, show that L'Hospital's rule fails for complex-valued functions.
11. a) If f is continuous on $[a, b]$, then prove that $f \in R(\alpha)$ on $[a, b]$.
 b) If f is monotonic on $[a, b]$ and if α is continuous on $[a, b]$, then prove that $f \in R(\alpha)$ on $[a, b]$.



Answer **any three** questions from this Part. **Each** question carries **13** marks. **(3×13=39)**

12. Let P be a non-empty perfect set in \mathbb{R}^k . Then prove that P is uncountable. Also prove that every interval is uncountable.
13. a) Define compact sets. Suppose $K \subset Y \subset X$. Then prove that K is compact relative to X if and only if K is compact relative to Y .
 b) Prove that compact subsets of metric spaces are closed.
14. a) Let f be monotonically increasing on (a, b) . Then prove that $f(x_+)$ and $f(x_-)$ exist at every point x of (a, b) and $\sup_{a < t < x} f(t) = f(x_-) \leq f(x) \leq f(x_+) = \inf_{x < t < b} f(t)$. Also if $a < x < y < b$, then prove that $f(x_+) \leq f(y_-)$.
 b) Prove that monotonic functions have no discontinuities of the second kind.
 c) Let f be monotonic on (a, b) . Then prove that the set of points of (a, b) at which f is discontinuous is at most countable.
15. State and prove Taylor's theorem.
16. Suppose $f \in R(\alpha)$ on $[a, b]$, $m \leq f \leq M$, ϕ is continuous on $[m, M]$ and $h(x) = \phi(f(x))$ on $[a, b]$. Then prove that $h \in R(\alpha)$ on $[a, b]$.