K25P 3578

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBCSS – OBE – Reg./Supple./Imp.) Examination, October 2025
(2023 Admission Onwards)
MATHEMATICS
MSMAT01C03 : Real Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any five** questions from this Part. **Each** question carries **4** marks. **(5×4=20)**

1. Prove that a set E is open if and only if its complement is closed.
2. Define a metric space. Give an example.
3. Define continuous mapping. Prove that the mapping $x \to |x|$ is continuous on R^k.
4. Let f be defined on [a, b]. If f is differentiable at a point $x \in [a, b]$, then show that f is continuous at x. Is the converse true ? Justify.
5. Suppose f is differentiable in (a, b). If $f'(x) = 0$ for all $x \in (a, b)$, then prove that f is a constant.
6. If $f_1 \in R(\alpha)$ on [a, b], and c is a constant, then prove that $cf_1 \in R(\alpha)$ and $\int_a^b cf_1 d\alpha = c\int_a^b f_1 d\alpha$.

PART – B

Answer **any three** questions from this Part. **Each** question carries **7** marks. **(3×7=21)**

7. Prove that the set of all rational numbers is countable.
8. Let f and g be complex continuous functions on a metric space X. Then prove that f + g, fg and $\frac{f}{g}$ are continuous on X.

P.T.O.

9. Give an example of a monotonic function f on (a, b) which is discontinuous at every point of a countable subset E of (a, b) and at no other point of (a, b). Justify.

10. By giving suitable example, show that L'Hospital's rule fails for complex-valued functions.

11. a) If f is continuous on [a, b], then prove that $f \in R(\alpha)$ on [a, b].

    b) If f is monotonic on [a, b] and if α is continuous on [a, b], then prove that $f \in R(\alpha)$ on [a, b].

PART – C

Answer **any three** questions from this Part. **Each** question carries **13** marks. **(3×13=39)**

12. Let P be a non-empty perfect set in R^k. Then prove that P is uncountable. Also prove that every interval is uncountable.

13. a) Define compact sets. Suppose $K \subset Y \subset X$. Then prove that K is compact relative to X if and only if K is compact relative to Y.

    b) Prove that compact subsets of metric spaces are closed.

14. a) Let f be monotonically increasing on (a, b). Then prove that f(x+) and f(x–) exist at every point of x of (a, b) and $\sup_{a<t<x} f(t) = f(x-) \leq f(x) \leq f(x+) = \inf_{x<t<b} f(t)$. Also if $a<x<y<b$, then prove that $f(x+) \leq f(y-)$.

    b) Prove that monotonic functions have no discontinuities of the second kind.

    c) Let f be monotonic on (a, b). Then prove that the set of points of (a, b) at which f is discontinuous is at most countable.

15. State and prove Taylor's theorem.

16. Suppose $f \in R(\alpha)$ on [a, b], $m \leq f \leq M$, ϕ is continuous on [m, M] and $h(x) = \phi(f(x))$ on [a, b]. Then prove that $h \in R(\alpha)$ on [a, b].